Analysis of Double Groove Guide Coupling Structures Using Domain Bases EW—BIEM

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***Abstract*—Double groove guide structures can be used to realize the coupling between groove guides of different kinds. In this letter, a novel method for studying these structures is introduced. The calculation model developed from eigen-weighted boundary integral equation method is given and applied to analyze double circular groove guide and circular-rectangular double groove guide. The numerical results are obtained and discussed.**

I. Introduction

BECAUSE of the recent advances in high-power relativistic electron beam oscillators in the frequency spectrum from short millimeter to submillimeter waves, new transmission media of good propagation characteristics are required. Groove guides are good choices. We have put forward a new type of groove guide circular groove guide. Our theoretical and experimental studies have shown that the circular groove guide has advantages of low loss, low dispersion, wide band, large dimensions, single-mode operation, and high-power handling capacity, and, in addition, it may be transformed to gyrotrons more convenient than other types of groove guide [1]. In the millimeter-wave system based on the circular groove guide, the coupling between two circular groove guides are often met. On the other hand, it is convenient to use conventional rectangular waveguide test bench and equipment for measuring the performances of circular groove guide components or circuits since there are no waveguide test bench and oscillators based on circular groove guides in the laboratory. So, the coupling problem between circular groove guides and rectangular waveguides should also be solved. Because of the open structure property of groove guides, double groove guide structures shown in Fig. 1 can be used to realize the coupling between circular groove guides and between circular groove guide and rectangular groove guide, respectively [2], [3]. J. M. Ruddy has designed a transducer from rectangular waveguide to a rectangular groove guide [4]. With the help of the transducer and the structure shown in Fig. 1(b), the coupling between the circular groove guide and the rectangular waveguide can also be achieved. The double groove guide may be considered as a system of two uniformly coupled groove guides. In order to understand its coupling performances, an exact analysis of the dominant TE even mode (TE_{11}^e) and TE odd mode (TE_{11}^o) is needed. In this letter, the domain bases eigen-weighted boundary integral equation method developed from EW—BIEM [5] is presented. By introducing the absorbing boundary condition, choosing weighting function, and expressing unknown function in terms of domain bases, the general calculation model is built, with which the eigenvalues of double groove guides are calculated. The numerical results are discussed and compared with the results obtained before.

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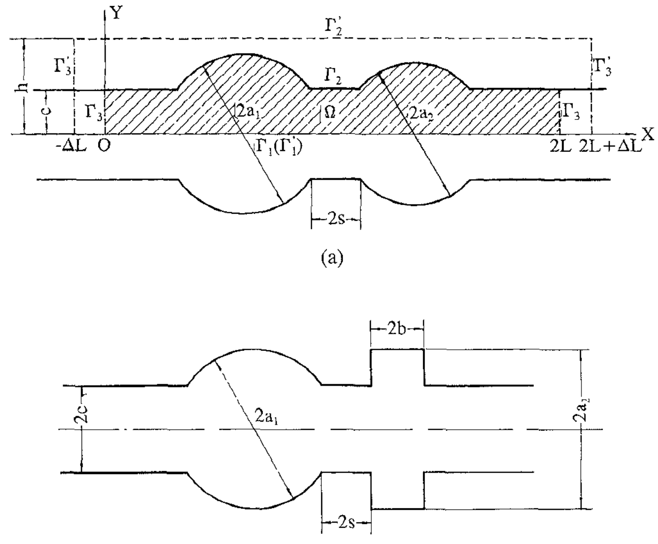

(a)

(b)

Fig. 1. The cross-section of double groove guides. (a) Double circular groove guide. (b) Circular-rectangular double groove guide.

II. Theoretical Analysis

A. Integral Equation

Let us consider the deterministic problem

$$\begin{cases} (\nabla_\perp^2 + k_c^2)u = 0 \\ \left(au + b\frac{\partial u}{\partial n}\right)_\Gamma = 0 \end{cases}$$

where u is the complex amplitude H_z or E_z in the guide with arbitrary cross-section for TE modes and TM modes, respectively, k_c is eigenvalue, $\nabla_\perp^2$ is 2-D Laplacian operator, and $\left(au + b\frac{\partial u}{\partial n}\right)_\Gamma = 0$ is the boundary condition of different kinds. By supposing the unknown u to be

$$u = u^{[N]} = \sum_{j=1}^{N} a_j \varphi_j \tag{1}$$

where φ_j is eigenfunction $(j = 1, 2, \cdots, N)$ and forcing the weighting function w_i to satisfy the following eigenvalue equation:

$$(\nabla_\perp^2 + k_{ci}^2) w_i = 0 \tag{2}$$

in which eigenvalue k_{ci} has been known, the integral equation of domain bases EW—BIEM may be reduced from the integral equation of weighted residuals method. It can be written as

$$\begin{aligned} &\int\int_\Omega \left[(k_c^2 - k_{ci}^2) u^{[N]} w_i \right] ds \\ &= \int_{\Gamma_2+\Gamma_3} u^{[N]} \frac{\partial w_i}{\partial n} dl - \int_{\Gamma_1} w_i \frac{\partial u^{[N]}}{\partial n} dl + \int_{\Gamma_3} a w_i u^{[N]} dl \end{aligned} \tag{3}$$

where $\alpha = \frac{a}{b}$ and Ω is the cross-section of the guide.

B. Calculation Model

We take the double circular groove guide shown in Fig. 1(a) as an example. Because the asymmetric structure help improve bandwidth, we are interested in the asymmetric double circular groove guide. According to its symmetry, only half of the cross-section should be considered. For dominant TE modes the symmetric plane OX is a magnetic wall and the guide wall is an electric wall. It is supposed that the parallel planes extend to infinite, but in the analysis of EW—BIEM these planes have to be cut at certain distances. Because there are no reflecting waves at the cut-off boundaries (Γ_3), the absorbing boundary condition should be satisfied.

Thus, the deterministic problem for solving dominant TE modes in the double circular groove guide can be expressed as

$$\begin{cases} (\nabla_\perp^2 + k_c^2) H_z = 0 \\ H_z|_{\Gamma_1} = 0 \\ \frac{\partial H_z}{\partial n}\Big|_{\Gamma_2} = 0 \\ \left(\frac{\partial H_z}{\partial n} + a H_z\right)\Big|_{\Gamma_3} = 0 \qquad \left(a = \sqrt{\left(\frac{\pi}{2c}\right)^2 - k_c^2}\right). \end{cases} \tag{4}$$

In order to simplify the boundary integral equation (3), the weighting function should satisfy the boundary condition as far as possible. So, four fictitious regular boundaries Γ_1', Γ_2' and two Γ_3' are made in which Γ_1' coincides with Γ_1'. The rectangular area enclosed by Γ_1', Γ_2', and Γ_3' includes the domain Ω. Let the weighting function w_i to be the eigenfunction of the following deterministic problem:

$$\begin{cases} (\nabla_\perp^2 + k_{cpq}^2) w_{pq} = 0 \\ w_{pq}\big|_{\Gamma_1'} = 0 \\ \frac{\partial w_{pq}}{\partial n}\Big|_{\Gamma_2'} = 0 \\ w_{pq}|_{\Gamma_3'} = 0 \end{cases} \tag{5}$$

then we have

$$w_i = w_{pq} = \sin\left[\frac{p\pi}{2L_x}(x + \triangle L)\right] \sin\left[\frac{(2q+1)\pi}{2h} y\right] \tag{6}$$

where

$$L_x = L + \triangle L, k_{cpq}^2 = \left(\frac{p\pi}{2L_x}\right)^2 + \left(\frac{2q+1}{2h}\pi\right)^2$$
$$(p = 1, 2, \cdots, q = 0, 1, 2, \cdots)$$

Making

$$H_z = H_z^{[N]} = \sum_{j=1}^{N} a_j \varphi_j$$

$$\varphi_j = w_j = \varphi_{mn} = \sin\left[\frac{m\pi}{2L_x}(x + \triangle L)\right] \sin\left[\frac{(2n+1)\pi}{2h} y\right]$$
$$(m = 1, 2, \cdots, n = 0, 1, 2, \cdots) \tag{7}$$

and noticing $w_i|_{\Gamma_1} = 0$, we obtain linear coupled equations expressed by matrix from (3). It reads

$$[A_{ij}(k_c)]\overline{a} = 0 \tag{8}$$

in which $\overline{a} = [a_1, a_2, \cdots, a_j, \cdots, a_N]'$

$$A_{ij}(k_c) = S_1(i,j) + \alpha S_2(i,j) - (k_c^2 - k_{cpq}^2) S_3(i,j)$$

$$S_1(i,j) = \int_{\Gamma_2+\Gamma_3} \varphi_j \frac{\partial w_i}{\partial n} dl$$

$$\begin{aligned} S_2(i,j) &= \int_{\Gamma_3} w_i \varphi_j dl \\ &= \int_0^c w_i(0,y)\varphi_j(0,y)\, dy + \int_0^c w_i(2L,y)\varphi_j(2L,y)\, dy \end{aligned}$$

$$S_3(i,j) = \int\int_\Omega w_i \varphi_j ds.$$

If $\overline{a}$ has nontrivial solution, there must be

$$\det[A_{ij}(k_c)] = 0. \tag{9}$$

Equation (9) is the eigenvalue equation of double circular groove guide, from which the eigenvalue k_c can be obtained, and it can also be used to analyze circular-rectangular double groove guide and other types of double groove guides provided that the boundary Γ_2 is the guide wall of the corresponding structures. Because the parameters S_1, S_2 and S_3 are unrelated to unknown k_c, they can be determined first and used directly in the iteration of k_c. By this way the computing speed is raised.

III. Numerical Results

By means of the calculation model given in Section II, the coupling structures of double circular groove guide and circular-rectangular double groove guide are calculated. Fig. 2 shows the theoretical curves of the relation between relative cutoff wavelength $\lambda_c/2c$ for TE_{11}^e mode and TE_{11}^o mode and relative groove spacing $2s/2c$. For small $2s/2c$, the difference between the even mode and odd mode is very obvious, however with the increment of $2s/2c$ the difference decreases, and finally the curves of TE_{11}^e mode and TE_{11}^o mode tend, respectively, to two limit values that are actually the same as the cutoff wavelengths obtained from [1] and [6] for the corresponding single groove guides (see Table I). This coincides with the fact that the larger the groove separation, the weaker the coupling between them, and as a result the double groove guide can be considered as two independent

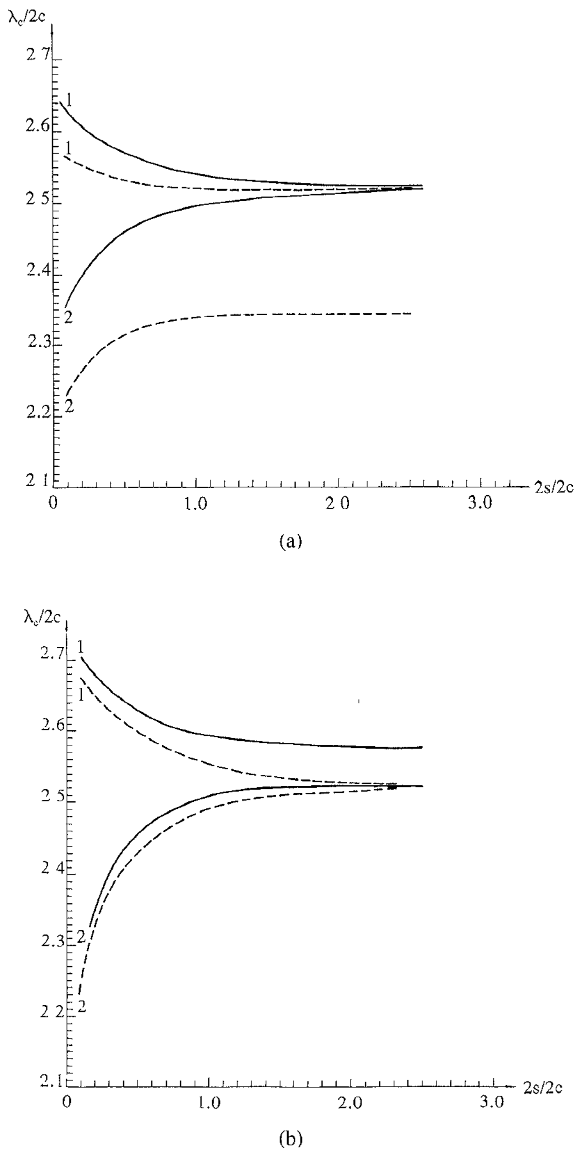

Fig. 2 The relation between relative cutoff wavelength $\lambda_c/2c$ and relative groove separation $2s/2c$. (a) Double circular groove guide (· ·: $a_2/a_1 = 1.08, 2c/2a_2 = 0.65$; ——· $a_2/a_1 = 1.0, 2c/2a_1 = 0.65$) (b) Circular-rectangular double groove guide (——: $a_2/a_1 = 1.08, a_2/b = 3.5, 2c/2a_1 = 0.65$, · · $a_2/a_1 = 1.08, a_2/b = 4, 2c/2a_1 = 0.65$). 1: TE^0_{11} mode; 2: TE^c_{11} mode.

TABLE I
THE CUTOFF WAVELENGTHS OF SINGLE GROOVE GUIDE

circular groove guide	2c/2a	$\lambda_c/2c$
	0.650	2.521
	0.702	2.348
rectangular groove guide (2c/2a=0.702)	a/b	$\lambda_c/2c$
	3.5	2.576
	4.0	2.529

single groove guides so long as the groove separation is large enough. The agreement of the limit values in Fig. 2 with the cutoff wavelengths in Table I also implies that our numerical results are reliable.

IV. CONCLUSION

The domain bases EW—BIEM is applied to analyze the coupling structures of the double groove guide. Our theoretical calculation shows that this method is an effective approach that is of high accuracy. The calculation model given in this letter is a general mode that can be used to calculate different double groove guide structures. Because both the weighting function and the base function are chosen as an known function, some parameters become constants and do not need to be calculated again in the process of iteration. In this way, the calculation time can be shortened.

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